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Improving Temporal Knowledge Graph Completion via Tensor Decomposition with Relation-Time Context and Multi-Time Perspective

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- Baselines models
- Fusing relation-time context and time properties
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- Fusion score function
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Introduction

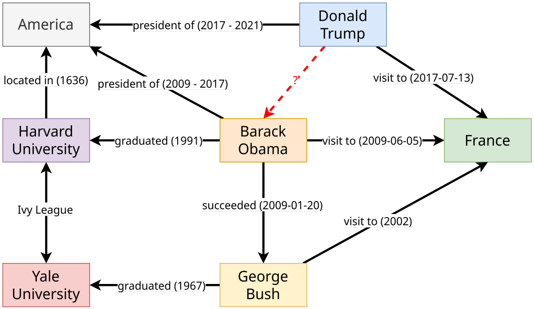

Figure: Example of Part of Temporal Knowledge Graph.

- In practice, data change over time.
- Reasoning problem on Temporal Knowledge Graph (TKG) can viewed in two settings:
  - Interpolation which focusing on completing the missing links at past timestamps.
  - Extrapolation which focusing on forecasting future facts.

$\Rightarrow$ We mainly focus on interpolation setting.

Challenges

General, when compared to extrapolation, interpolation setting is more easier, but still faces many challengs:

- The flexibility of these models is limited;
- Not utilizing temporal information to improve the quality of learning embedding during model training;
- The connection between relations and the timestamps attached to them has not been fully exploited.

Our contributions

- We propose MPComplEx, a novel temporal knowledge graph completion model using tensor decomposition and weighted feature combination.
- MPComplEx introduces entity-specific weights and controlled temporal embeddings to capture multiple time perspectives.
- Modelling relation-timestamp correlation via a dot product, enhancing relational embedding quality.
- Experiments on benchmark datasets show significant improvements in link prediction.

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Basic Notations

- Temporal knowledge graph $\mathcal{G} = \{\mathcal{Q}, \mathcal{E}, \mathcal{R}, \mathcal{T}\}$
- A quadruplet is denoted as (s, r, o, t).

We focus on TKGC tasks which often considered as predicting missing entities in a given data set like $(s, r, ?, t)$ where $?$ denotes the missing element.

Low-Rank Decomposition

Given a tensor $\mathcal{X} \in \mathbb{R}^{n_1 \times n_2 \times n_3}$. let $a_i \in \mathbb{R}^{n_1}$, $b_i \in \mathbb{R}^{n_2}$, and $c_i \in \mathbb{R}^{n_3}$ represent the component vectors of a rank-one tensor, and let $R > 0$ denote the rank of the CP decomposition. The CP decomposition can be expressed as follows:

$$\forall(i, j, k), \mathcal{X}_{i,j,k} \approx \sum_{r=1}^{R} a_{i,r} \circ b_{j,r} \circ c_{k,r} = \langle a_i, b_j, c_k \rangle \tag{1}$$

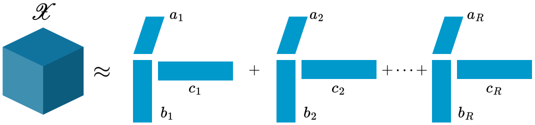

Figure: The visualization of Canonical polyadic (CP) decomposition on three-way tensor.

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Previous TKG Interpolation models

Model	Score function	Simultaneousness	Aggregation	Associativity
TTransE	$\|\mathbf{C_s} + \mathbf{C_r} + \mathbf{C_t} - \mathbf{C_o}\|$	x	x	x
HyTE	$\|P_t(\mathbf{C_s}) + P_t(\mathbf{C_r}) - P_t(\mathbf{C_o})\|$	x	x	x
TeRo	$\|\mathbf{C_s} \circ \mathbf{C_t} + \mathbf{C_r} - \mathbf{C_o} \circ \mathbf{C_t}\|$	✓	x	x
RotatQVS	$\|\mathbf{C_s}\mathbf{C_t}^{-1} + \mathbf{C_r} - \mathbf{C_t}\mathbf{C_o}^{-1}\|$	✓	x	x
TA-DistMult	$\langle \mathbf{C_s}, \text{LSTM}([\mathbf{C_r}; \mathbf{C_t}^{seq}]), \mathbf{C_o}\rangle$	x	x	x
TComplEx	$\text{Re}\left(\langle \mathbf{C_s}, \mathbf{C_r}, \overline{\mathbf{C_o}}, \mathbf{C_t}\rangle\right)$	x	x	x
TimePlex	$\langle \mathbf{C_s}, \mathbf{C_r}^{SO}, \mathbf{C_o}\rangle + \alpha\langle \mathbf{C_s}, \mathbf{C_r}^{ST}, \mathbf{C_t}\rangle$	✓	✓	x
ChronoR	$\langle \mathbf{C_s} \circ [\mathbf{C_r} \vert \mathbf{C_t} \vert \mathbf{C_o}]\rangle$	✓	x	x
ST-ConvKB	$\text{contact}\left(g([\mathbf{C_s} : \mathbf{C_r} : \mathbf{C_t}] * \Omega)\right) \cdot \mathbf{w}$	✓	x	x
TPComplEx	$\text{Re}\left(\langle \mathbf{C_s} + \mathbf{C_{t2}}, \mathbf{C_r}, \mathbf{C_o} + \mathbf{C_{t3}}, \mathbf{C_{t1}}\rangle\right)$	✓	✓	✓
MPComplEx (Ours)	$\text{Re}\left(\langle C_{st_2}, C_{rc}, \overline{C_{ot_3}}, C_{t_1}\rangle\right)$	✓	✓	✓

Table: Comparison of Temporal Knowledge Graph Models

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Baseline models: TComplEx and TPComplEx

TComplEx score function can be formulated as follows:

$$\phi(s, r, o, t) = \text{Re}\left(\langle C_s, C_r, \overline{C_o}, C_t \rangle\right), \tag{2}$$

where $\phi()$ denotes the scoring function, Re$(.)$ returns the real vector component for input embedding; $C_s, C_r, C_o, C_t \in \mathbb{R}^{2\times d}$ denotes the complex embedding with embedding rank d for subject, relation, object, and timestamp, respectively.

Baseline models: TComplEx and TPComplEx

TPComplEx score function can be formulated as follows:

$$\phi(s, r, o, t) = \mathrm{Re}\left(\left\langle C_s + C_{t_2}, C_r, \overline{C_o + C_{t_3}}, C_{t_1} \right\rangle\right), \tag{3}$$

where C_{t_1} is the temporal embedding, and C_{t_2}, C_{t_3} are additional temporal embedding for subject and object, respectively.

Control score function

Let $\alpha_1, \alpha_2 \in \mathbb{R}^+ \cup \{0\}$ and $\beta_1, \beta_2 \in \mathbb{R}^+ \cup \{0\}$. We apply these variable for subject and object embedding, respectively. Consequently, the TPComplEx score function becomes:

$$\begin{aligned} \phi_1(s, r, o, t) = \phi_{\text{base}}(s, r, o, t) + G_r^{t_2}(C_{t_2}) \\ + G_r^{t_3}(C_{t_3}) + G_r^{t_4}(C_{t_4}), \end{aligned} \tag{4}$$

where $\phi_{\text{base}}(s, r, o, t) = \text{Re}\left(\langle \alpha_1 C_s, C_r, \beta_1 \overline{C_o}, C_{t_1} \rangle\right)$, C_{t_1} represents the temporal embedding, while $C_{t_2}, C_{t_3}, C_{t_4}$ denote additional temporal embeddings.

Control score function

Then, we define C_{t_2}, C_{t_3} as:

$$G_r^{t_2}(C_{t_2}) = \mathsf{Re}\left(\left\langle C_r, \overline{C_o}, C_{t_1}, \alpha_2 C_{t_2}\right\rangle\right), \tag{5}$$

$$G_r^{t_3}(C_{t_3}) = \mathsf{Re}\left(\left\langle C_s, C_r, C_{t_1}, \beta_2 \overline{C_{t_3}}\right\rangle\right). \tag{6}$$

And C_{t_4}:

$$G_r^{t_4}(C_{t_4}) = \mathsf{Re}\left(\left\langle C_r, C_{t_1}, \alpha_2 C_{t_2}, \beta_2 \overline{C_{t_3}}\right\rangle\right). \tag{7}$$

Thus, we obtain a new score function, which is defined as:

$$\begin{aligned} \phi_1(s,r,o,t) &= \phi_{\text{base}}(s,r,o,t) \\ &+ \mathsf{Re}\left(\left\langle C_r, \beta_1 \overline{C_o}, C_{t_1}, \alpha_2 C_{t_2}\right\rangle\right) \\ &+ \mathsf{Re}\left(\left\langle \alpha_1 C_s, C_r, C_{t_1}, \beta_2 \overline{C_{t_3}}\right\rangle\right) \\ &+ \mathsf{Re}\left(\left\langle C_r, C_{t_1}, \alpha_2 C_{t_2}, \beta_2 \overline{C_{t_3}}\right\rangle\right) \end{aligned} \tag{8}$$

Simplifying the above expression, we have:

$$\phi_1(s,r,o,t) = \mathsf{Re}\left(\left\langle C_{st_2}, C_r, \overline{C_{ot_3}}, C_{t_1}\right\rangle\right), \tag{9}$$

where $C_{st_2} = \alpha_1 C_s + \alpha_2 C_{t_2}$, $C_{ot_3} = \beta_1 C_o + \beta_2 C_{t_3}$.

Define relation-time interactions

We define relation-time interactions as dot product:

$$C_{rt} = \langle C_r, C_t \rangle \tag{10}$$

Then, replace C_r by C_r, we obtain:

$$\begin{aligned} \phi_2(s,r,o,t) = \phi_{\textsf{base}}(s,r,o,t) + G_r^{t_2}(\alpha_2 C_{t_2}) \\ + G_r^{t_3}(\beta_2 C_{t_3}) + G_r^{t_4}(C_{t_4}). \end{aligned} \tag{11}$$

By expanding the above score function similar to Eq. 8, we have the score function when using relation-time context embedding as follows:

$$\phi_2(s,r,o,t) = \mathrm{Re}\left(\left\langle C_{st_2}, C_{rt}, \overline{C_{ot_3}}, C_{t_1} \right\rangle\right), \tag{12}$$

Fusion score function

By combining the weighted score functions from Eq. 12 and Eq. 9, we derive a more generalized score function, which is formulated as follows:

$$\begin{aligned}\phi(s,r,o,t) &= \phi_1(s,r,o,t) + (1-\gamma)\phi_2(s,r,o,t) \\ &= \mathrm{Re}\left(\langle C_{st_2}, C_{rc}, \overline{C_{ot_3}}, C_{t_1}\rangle\right),\end{aligned} \tag{13}$$

where the combined embedding C_{rc} is defined as $C_{rc} = \gamma C_r + (1-\gamma)C_{rt}$. Here, γ is the weight factor that balances the percentage of features of the relation derived from C_r and the relation-time context embedding C_{rt}.

Loss function

We compute the instantaneous multi-class loss for each training quadruple (s, r, o, t) as follows:

$$\mathcal{L} = -\phi(s, r, o, t) + \log \left[\sum_{\substack{o' \neq o \\ o' \in \mathcal{E}}} \exp\left(\phi(s, r, o', t)\right) \right], \tag{14}$$

where $\phi(.)$ represents the score function. In addition, we include a regularization term, $\mathcal{L}_{reg}$. Therefore, the final loss function used for training is given by:

$$\mathcal{L}_{\text{total}} = \mathcal{L} + \mathcal{L}_{reg}. \tag{15}$$

Regularization

Similarly to TPComplEx [3], our model incorporates entities with temporal bias into the regularization process and adopts N3 regularization as defined by [10]. The regularization function is expressed as:

$$\mathcal{L}_{reg} = \lambda_1 \left(\|C_{st_2}\|_3^3 + \|C_{rc}\|_3^3 + \|C_{ot_3}\|_3^3 \right) + \lambda_2 \|C_t\|_3^3 , \tag{16}$$

where λ_1 and λ_2 are the regularization weights for the entity-relation and temporal embeddings, respectively.

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Datasets and Baselines

- Baselines.: TTransE [12]; TComplEx, TNTComplEx [10]; TimePlex [9]; ChronoR [7]; TeLM [8]; BTDG [6]; TBDRI [5]; SANe [1]; MTComplEx [4]; TPComplEx [3]; MvTuckER [2].
- Datasets. ICEWS14, ICEWS05-15, YAGO15k [11], and GDELT [13].

Performance Comparision

Model	MRR ↑	Hit@1 ↑	Hit@10 ↑	MRR ↑	Hit@1 ↑	Hit@10 ↑
	ICEWS14			ICEWS05-15		
TTransE [12]	0.255	0.074	0.601	0.271	0.084	0.616
TComplEx [10]	0.610	0.530	0.770	0.660	0.590	0.800
ChronoR [7]	0.625	0.547	0.773	0.675	0.596	0.820
TeLM [8]	0.625	0.545	0.774	0.678	0.599	0.823
BTDG [6]	0.601	0.516	0.753	0.627	0.534	0.798
TBDRI [5]	0.652	0.552	0.785	0.709	0.646	0.821
SANe [1]	0.638	0.558	0.782	0.683	0.605	0.823
MTComplEx [4]	0.629	0.548	0.782	0.675	0.592	0.822
TPComplEx [3]	<u>0.898</u>	<u>0.865</u>	<u>0.954</u>	<u>0.845</u>	<u>0.794</u>	<u>0.934</u>
MvTuckER [2]	0.654	0.577	0.797	0.698	0.618	0.841
MPComplEx (Ours)	**0.941**	**0.931**	**0.957**	**0.962**	**0.957**	**0.974**
APG (%) ↑	**4.30**	**6.60**	**0.30**	**11.70**	**16.30**	**4.00**
RPG (%) ↑	**4.79**	**7.63**	**0.31**	**11.48**	**17.38**	**2.78**

Performance Comparision

Model	MRR ↑	Hit@1 ↑	Hit@10 ↑	MRR ↑	Hit@1 ↑	Hit@10 ↑
	YAGO15k			GDELT		
TTransE [12]	0.321	0.230	0.510	0.115	0.000	0.318
TComplEx [10]	0.360	0.280	0.540	0.298	0.213	0.464
ChronoR [7]	0.366	0.292	0.538	-	-	-
TBDRI [5]	0.368	0.301	0.554	0.269	0.164	0.441
SANe [1]	-	-	-	0.301	0.212	0.476
TPComplEx [3]	<u>0.689</u>	<u>0.651</u>	<u>0.762</u>	0.407	0.329	0.559
MvTuckER [2]	-	-	-	<u>0.549</u>	<u>0.477</u>	<u>0.682</u>
MPComplEx (Ours)	**0.904**	**0.899**	**0.914**	**0.676**	**0.630**	**0.762**
APG (%) ↑	**21.50**	**24.80**	**15.20**	**26.90**	**30.1**	**20.30**
RPG (%) ↑	**31.20**	**38.10**	**19.95**	**66.09**	**91.49**	**36.31**

Computational Complexity

Table: Model parameters and embedding ranks for our baselines and proposed models.

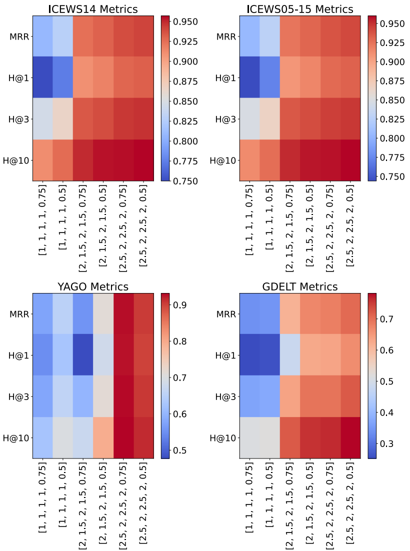
Models	Parameters	ICEWS14	ICEWS05-15	YAGO15k	GDELT
ComplEx	$2d(\lvert\mathcal{E}\rvert + 2\lvert\mathcal{R}\rvert)$	1820	860	1960	3820
TComplEx	$2d(\lvert\mathcal{E}\rvert + \lvert T\rvert + 2\lvert\mathcal{R}\rvert)$	1740	1360	1940	2270
TPComplEx	$2d(\lvert\mathcal{E}\rvert + 3\lvert\mathcal{T}\rvert + 2\lvert\mathcal{R}\rvert)$	1594	886	1892	1256
MPComplEx	$2d(\lvert\mathcal{E}\rvert + 3\lvert\mathcal{T}\rvert + 2\lvert\mathcal{R}\rvert)$	1500	800	1500	1200

Analysis of the effects of relation-time context features

Table: The influence of relation-time context features on the datasets.

Case study	MRR ↑	Hit@1 ↑	Hit@10 ↑	MRR ↑	Hit@1 ↑	Hit@10 ↑
	ICEWS14			ICEWS05-15		
Only relation-time	0.923	0.912	0.940	0.930	0.920	0.948
W/o relation-time	0.920	0.910	0.940	0.946	0.939	0.963
Fusion features	0.941	0.931	0.957	0.962	0.957	0.974
	YAGO15k			GDELT		
Only relation-time	0.794	0.774	0.834	0.719	0.680	0.793
W/o relation-time	0.774	0.752	0.815	0.719	0.680	0.793
Fusion features	0.904	0.899	0.914	0.676	0.630	0.762

Analysis of the effects of weight combinations



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- Introduces a Multi-Time Perspective Relation-Time Context ComplEx Embedding model.
- Enhances flexibility with adjustable temporal embeddings.
- Maintains computational efficiency with a fixed parameter count.
- Integrates relation-timestamp correlations into the scoring function.
- Improves relation embeddings and prediction accuracy.
- Future work: exploring cross-temporal patterns and extrapolation challenges.

Thanks for your attention

References I

[1] Yancong Li et al. "SANe: Space adaptation network for temporal knowledge graph completion". In: *Information Sciences* 667 (2024), p. 120430.

[2] Hao Wang et al. "MvTuckER: Multi-view knowledge graphs representation learning based on tensor tucker model". In: *Information Fusion* 106 (2024), p. 102249.

[3] Jinfa Yang et al. "Tensor decompositions for temporal knowledge graph completion with time perspective". In: *Expert Systems with Applications* 237 (2024), p. 121267.

[4] Fu Zhang et al. "Joint framework for tensor decomposition-based temporal knowledge graph completion". In: *Information Sciences* 654 (2024), p. 119853.

[5] Mei Yu et al. "TBDRI: block decomposition based on relational interaction for temporal knowledge graph completion". In: *Applied Intelligence* 53.5 (2023), pp. 5072–5084.

[6] Yujing Lai et al. "Block term decomposition with distinct time granularities for temporal knowledge graph completion". In: *Expert Systems with Applications* 201 (2022), p. 117036.

References II

[7] Ali Sadeghian et al. "Chronor: Rotation based temporal knowledge graph embedding". In: *Proceedings of the AAAI conference on artificial intelligence*. 2021, pp. 6471–6479.

[8] Chengjin Xu et al. "Temporal knowledge graph completion using a linear temporal regularizer and multivector embeddings". In: *Proceedings of the 2021 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies*. 2021, pp. 2569–2578.

[9] Prachi Jain, Sushant Rathi, Soumen Chakrabarti, et al. "Temporal Knowledge Base Completion: New Algorithms and Evaluation Protocols". In: *Proceedings of the 2020 Conference on Empirical Methods in Natural Language Processing (EMNLP)*. 2020, pp. 3733–3747.

[10] Timothée Lacroix, Guillaume Obozinski, and Nicolas Usunier. "Tensor Decompositions for Temporal Knowledge Base Completion". In: *International Conference on Learning Representations*. 2019.

References III

[11] Alberto García-Durán, Sebastijan Dumancic, and Mathias Niepert. "Learning Sequence Encoders for Temporal Knowledge Graph Completion". In: *Conference on Empirical Methods in Natural Language Processing*. 2018.

[12] Julien Leblay and Melisachew Wudage Chekol. "Deriving Validity Time in Knowledge Graph". In: *Companion Proceedings of the The Web Conference 2018* (2018).

[13] Rakshit Trivedi et al. "Know-evolve: Deep temporal reasoning for dynamic knowledge graphs". In: *international conference on machine learning*. PMLR. 2017, pp. 3462–3471.